A Hough Transform Track Initiation Algorithm for Multiple Passive Sensors[1]

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Abstract: *This paper is concerned with the problem of associating measurements from several passive (angle-only) sensors. Contrast with the static problem when measurements at a given time are processed, we present here a technique, processing measurements from several consecutive scans. Here we use a particular feature of Hough Transform for batch data processing. The technique is derived assuming presence of clutter, missed detections and an unknown number of targets. An effective heuristic technique for ghost elimination is proposed here which preliminary reduces considerably the total number of so called ghost intersections and thus alleviating consequent Hough Transform algorithm implementation. This Hough Transform algorithm as it can be seen from numerical results presented here, initiates tracks and successfully resolves the ambiguous measurement-target associations. The main advantage of presented here approach is avoiding the S-D matching problem which is known to be NP-hard.*

Keywords: Data Fusion, Hough Transform, Passive Sensors.

1 Introduction

This paper concerns the use of Hough transform (HT) method for multisensor data association. Several passive (angle-only) sensors observe common surveillance area. The sensors synchronously send measurements of unknown number of targets at one and the same moment of time. The corresponding Data Association (DA) problem is usually considered in two ways: static and dynamic. In the static DA problem the measurements propagated to a common time are clustered into classes, where each class belongs to particular target. The optimal clustering maximizes a corresponding likelihood function. Many authors have investigated this problem [5,6].

In this paper the dynamic case is considered, when data from several consecutive scans are processed. The sampling intervals (periods of scans) are not necessarily equal. It is supposed that targets move rectilinearly and that the passive sensors are with known fixed positions. The target sites are obtained by triangulation of bearing measurements from at least two sensors. However, multiple targets create a number of false triangulations, called ghosts that can not be discarded on the base of information obtained from just two sensors. To overcome this problem it is necessary to use additional sensors to resolve ambiguous measurement-target associations. Unfortunately, for number of sensors more than or equal three the corresponding matching problem can be shown to be NP-complete (the so-called "combinatorial explosion" arises) [5]. A special attention deserves dense target scenario with presence of heavy clutter, when the most of the known algorithms dramatically reduce their efficiency.

Recently, a new DA approach using HT has been proposed for active sensors (radars) [4]. It successfully overcomes the combinatorial explosion. Here, one solution for DA problem using HT is developed in the case of passive sensors. A new transform equation is applied. This approach allows both to detect and initiate existing tracks and to reject false tracks, consisting of ghosts. A heuristic ghost elimination technique is also developed. It increases the probability of recognizing false tracks. The proposed HT algorithm improves the overall system performance and capabilities.

This paper is organized as follows. In Section 2 the problem formulation is given. The main pruning rules and heuristic ghost elimination technique are described in Section 3. Section 4 is devoted to data association algorithm, based on HT. The original HT is briefly described in Section 4.1. A presentation of HT as template matching technique is given in Section 4.2. The new transform equation is proposed in Section 4.3. In Section 4.4 we describe the philosophy of DA algorithm, based on HT, and his ability to detect and to reject ghosts. Section 5 provides some application results and Section 6 gives summary and conclusions of this work.

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2 Problem Formulation

Let suppose that several passive sensors track at least two targets in k consecutive scans. When missed detections and false alarms don't exist, the lists of reports from the sensors contain equal number of measurements. But in more common case, when false alarms and missed detections are assumed, the number of measurements varies from scan to scan. If the index S is used to denote the sensor number and t is used to denote the temporal dimension (the scan number) the measurements received in k consecutive scans can be denoted by: $z_1^S(t_1),...,z_{m_1}^S(t_1),z_1^S(t_2),...,z_{m_2}^S(t_2),...,z_1^S(t_k),...,z_{m_k}^S(t_k)$. Every measurement $z_i^S(t)$ can be expressed as a sum of true target bearing β_i^S measured from the "North" direction and the measurement error $v_i^S \sim N(0,\sigma_S^2)$ at the scan t :

$$z_i^S(t) = \beta_i^S + v_i^S \; . \qquad (1)$$

In this paper it is assumed that data synchronously arrives from the sensors. Thus, to obtain the target position it is necessary to triangulate bearing measurements arriving in one and the same frame from each pair of sensors. To illustrate the problem let consider two targets in common surveillance space without false alarms and missed detections. In this case exactly two measurements will be received from each sensor in a scan. The triangulation will give four potential positions for two existing targets (figure 1).

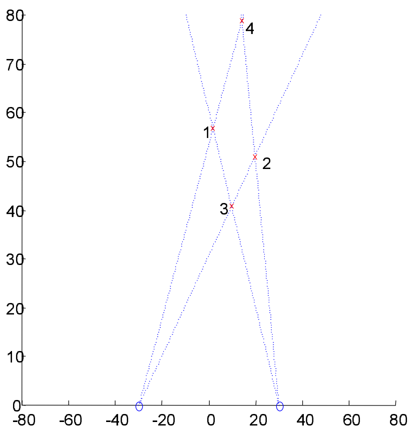

Figure 1. Two sensors observe two approaching targets

In dynamic case the measurements from several consecutive scans are processed simultaneously. As a result of triangulation tracks consisted of ghosts appear in parallel with real tracks. Correlation logic has to be applied to recognize and initiate the true rectilinear tracks and to reject the tracks consisting of ghosts.

3 A Preliminary Ghost Elimination

For the chosen scenario a heuristic technique for preliminary ghosts elimination is proposed. The basic idea can be illustrated by means of the most simple scenario in which two sensors observe two approaching targets (figure 1), in which case four triangulations are generated.

As we don't know which of these triangulations are the true targets it is necessary to create all feasible partitionings of them. This partitioning is one of the pruning rules of the eliminating technique. In our case the possible partitioning is (1,2) and (3,4). It is also obviously that partitioning (1,3) and (2,4) are unfeasible.

And now, if the difference $\Delta_{i,j} = z_i^K(t_l) - z_j^L(t_l)$, concerning triangulation 4, is too small this triangulation can be found to be out of a maximal range of sensor sensitivity D_{max}. Obviously, this triangulation has to be stated as a false one. But triangulation 3 has to be stated as a false intersection, too, because it is coupled with triangulation 4 in a feasible partitioning. A particular case of this rule is when $\Delta_{i,j} < 0$, i.e., the triangulation is in a half-space behind the sensors and it has to be stated as a false intersection.

In brief, the ghost rejection algorithm is based on the next two rules:

- Out-of-range elimination;
- No more than one target on a beam.

According these two rules the algorithm can be set out in two-sensor case as follows. Assume we have two lists of measurements received from two sensors in one scan: $z_1^K, z_2^K, \ldots, z_n^K$ and $z_1^L, z_2^L, \ldots, z_n^L$.

1. For each i and j $(i,j = \overline{1,n})$ the differences $\Delta_{i,j} = z_i^K - z_j^L$ are computed.

1.1. If the difference $\Delta_{i,j} \le 0$, the element of matrix $M(i,j)$ is set to zero;

1.2. If $\Delta_{i,j} > 0$, compute the ratios: $\dfrac{\cos z_i^K}{\sin \Delta_{i,j}}$ and $\dfrac{\cos z_j^L}{\sin \Delta_{i,j}}$;

1.2.1. If any of them exceeds the value of normalized range of sensors' sensitivity $\dfrac{D_{max}}{B_{K,L}}$, the element $M(i,j)$ is set to zero ($B_{K,L}$ denotes distance between corresponding sensors).

1.2.2. Otherwise, $M(i,j) = \Delta_{i,j}$.

The second part of the algorithm includes checking of the rows and columns of *M*.

2. Omitting already processed rows, check consecutively each row and count the non-zero elements in it.

2.1. If there is no one row with a single non-zero element, then Go To **2.5**.
2.2. If all rows are already processed, Go To **2.6.**
2.3. If find a row, say k, which single non-zero element is $M(k,l)$, set $M(i,l)=0$, for all $i \neq k$.
2.4. Mark the row k and column l as processed and Go To **2.**
2.5. Repeat all steps from **2** to **2.4**, but instead of rows, check columns.
2.6. End.

The example bellow illustrates the efficiency of this approach for a pair of passive sensors ($B_{K,L}=20\,km$) observing an area with size $-80km \div 80\,km$. As it is shown on figure 2, there are two real trajectories and a false one. The second false trajectory is placed out of the surveillance region and it is not displayed here. Implying this technique the true trajectories are recognized and the false one is rejected (figure 3).

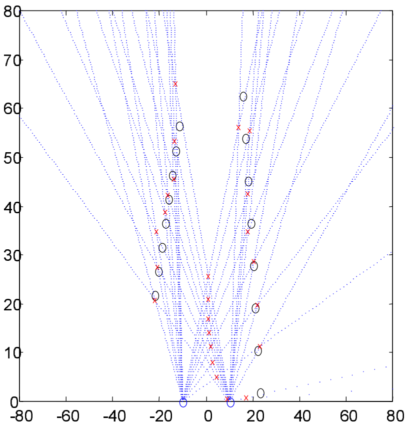

Figure 2. Two real trajectories and a false one

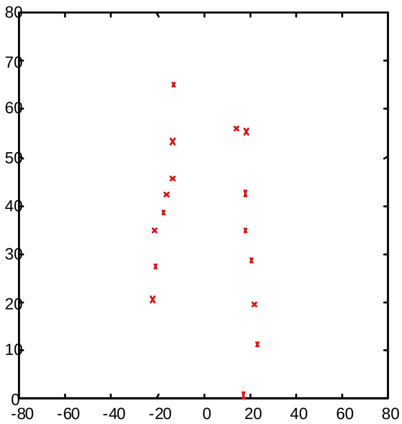

Figure 3. Recognizing of true trajectories

This preliminary technique does not solve the problem of ghosts elimination entirely, so another, more powerful approach, based on HT application is proposed below.

4 Measurement association algorithm based on Hough Transform

4.1 The original Hough Transform

HT was patented in 1962 [1] as a mathematical transformation of points from input space, referred to as feature space (FS) into curves in a special parameter space (PS) and was used for straight line detection. This method is based on the fact that, all points from a straight line positioned in FS can be mapped in a single point in PS. Many applications in image recognition use HT for detection of different image features - straight lines, circles, ellipses, and etc. [3]. The most commonly used mapping equation for straight line detection is so called normal HT equation [2]:

$$\rho = x\cos\theta + y\sin\theta\,. \qquad (2)$$

The algorithm maps each point (x,y) from FS to a curve in the PS (ρ,θ). For each value of the discrete parameter θ in (2) the corresponding values of parameter ρ (the line's shift from the coordinate center) are calculated. If several points in FS lie on a straight line the corresponding curves are intersected in a single point in PS. A simple "voting" algorithm is used to locate this point. For this purpose the PS is partitioned into set of accumulators each with size ($\Delta\rho$, $\Delta\theta$). Thus, the obtained above results can be considered as addresses of the accumulators, which contents are respectively increased by unity. A rectangular strip (template) in the FS corresponds to every accumulator from PS, so the accumulator parameters from PS can be considered as template parameters in FS. The peak of votes will occur in the accumulator which corresponding template includes the most part of points of a line.

One of the most valuable features of HT algorithm is that its computational complexity depends almost linearly on number of processed points.

4.2 Hough Transform as template matching technique

The presence of additive measurement noise (1) leads to spreading the triangulation points around the true straight line trajectory. This means that corresponding curves into PS will not intersect precisely in one point. In this way the HT acts as an estimator of the closeness of triangulations to the templates. A brief description of this feature is next given.

Let q is the parameter vector in PS and d_k^j denotes *j*-th intersection point received in a chosen coordinate system in FS (for example, $d_k^j=\left(x_k^j, y_k^j\right)$), in data frame k. A template of i-th track $r_i=r\left(d,q_i\right)$ can be determined for a particular value of q_i by the equation $r(d,q_i)=0$. The algorithm tests each received point using this expression. For a particular point d_k^j and template r_i the value $\zeta_i^j = r(d_k^j, q_i)$ is calculated. The degree of closeness of the point d_k^j to i-th checked template is expressed by ζ_i^j. If the decision threshold is denoted by Ω, the voting rule can be described as:

$$\Lambda_i^j(k) = \begin{cases} 1, & if\ \exists d_k^j \quad j = 1,\dots,m_k \quad for\ which \quad \left|\zeta_i^j\right| \le \Omega \\ 0, & if\ for\ \forall d_k^j \quad j = 1,\dots,m_k \quad \left|\zeta_i^j\right| > \Omega \end{cases}.$$

Here $\Lambda_i^j(k)$ expresses the result of voting procedure in i-th accumulator in frame k; m_k is the number of points in frame k. The magnitude of possible correlation between the points from any n consecutive frames and template q_i is expressed as:

$$\Lambda_i = \sum_{k=l+1}^{l+n} \sum_{j=1}^{m_k} \Lambda_i^j(k), \ l = 1,2,3,\dots$$

Suppose H_0 is hypothesis that specifies the target existence and H_1 is the alternative hypothesis. By the following test an inference can be taken about existence/absence of a target trajectory corresponding to template q_i:

$$\begin{aligned} H_0 &: \ if \quad \Lambda_i > g_{thres} \\ H_1 &: \ if \quad \Lambda_i \le g_{thres} \end{aligned}.$$

4.3 Transform Equation for Passive Sensors

The implementation of HT for rectilinear trajectory detection and for bearing-only sensors requires new form of mapping equation. An appropriate mapping equation for active sensors using measurements arriving in polar coordinates is proposed in [4]. It is modified here for passive sensors as follows:

$$\rho = \rho^K + \rho_{cor}^K = \rho^L + \rho_{cor}^L . \qquad (3)$$

In the transform equation corresponding terms are:

$$\rho^K = D_{i,j}^K \sin\left(\psi - z_i^K\right) = B_{K,L} \frac{\cos z_j^L}{\sin\left(z_i^K - z_j^L\right)} \sin\left(\psi - z_i^K\right),$$

$$\rho^L = D_{i,j}^L \sin\left(\psi - z_j^L\right) = B_{K,L} \frac{\cos z_i^K}{\sin\left(z_i^K - z_j^L\right)} \sin\left(\psi - z_j^L\right),$$

$$\rho_{cor}^K = D_K \cos(\psi - \alpha_K),$$

$$\rho_{cor}^L = D_L \cos(\psi - \alpha_L).$$

Here $D_{i,j}^K$ and $D_{i,j}^L$ are the target ranges computed by triangulation from sensor K and sensor L, respectively. The correction terms ρ_{cor}^K and ρ_{cor}^L transform results from local sensor coordinate system to absolute polar coordinate system and depend on the sensors' positions $\left(D_K, \alpha_K\right)$ and $\left(D_L, \alpha_L\right)$. Instead of angle θ in (2) the target heading $\psi = \theta + \frac{\pi}{2}$ is used here.

4.4 Ghost detection using Hough Transform

Hough Transform track detector discovers real tracks when two conditions are satisfied. The both conditions can be used successively to reject ghosts.

The first of them was discussed in Subsection 4.2. A set of measurements will be accepted as a track if these measurements are ordered on, or near to a straight line. Otherwise this set of measurements will be rejected as a track. In practice, the trajectories consisting of ghosts very often are not straight lines and they will be rejected even in the case of two sensors.

Sometimes, however, a set of ghost triangulations can meet this condition and will be accepted as a real trajectory. For this reason, template matching technique can only reduce the number of ghosts, but can not solve the problem entirely. The main advantage of this technique is that it can be applied in the simplest case when two passive sensors are used only.

The second condition is more powerful, but it assumes more than two sensors. If three or more sensors are used, the lines formed by ghosts will be confirmed (even if they satisfy the first condition) only by one pair of sensors, because ghosts' coordinates depend on sensor's positions - unique for each possible pair of sensors. For n sensors the real trajectory will be confirmed C_n^2 times at the same place, while the ghost trajectories will be detected at different locations depending on particular sensor pair. An example is demonstrated on figure 4 and figure 5 for 3 sensors.

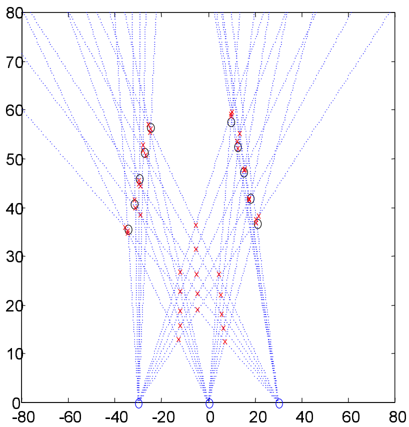

Figure 4. Three sensors observe two trajectories

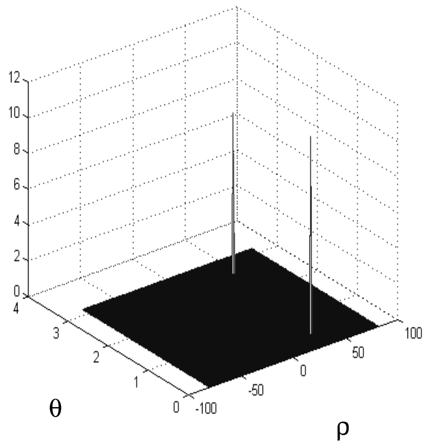

Figure 5. Parameter space for three sensor case

5 Application results

The proposed algorithms have been tested extensively on a variety of scenarios. On the figure 2, figure 6 and figure 7 the results from HT track initiation algorithm are presented. The results are received by standard HT algorithm without preliminary ghost elimination technique. The scenario includes two approaching targets and two sensors, with coordinates $(10,-\pi/2)$ and $(10,\pi/2)$. The speed of targets and their directions are randomly chosen. Sensor data are received in 8 scans without missed detections and false alarms. The real target positions are signed with “o” on figure 1. The symbol “x” is used to display the received noisy measurements (or corresponding to them triangulation points - real and ghosts). On the figure 6 the accumulator contents can be seen. The results from hypothesis test are shown on figure 7. There are several peaks, corresponding to real and false trajectories.

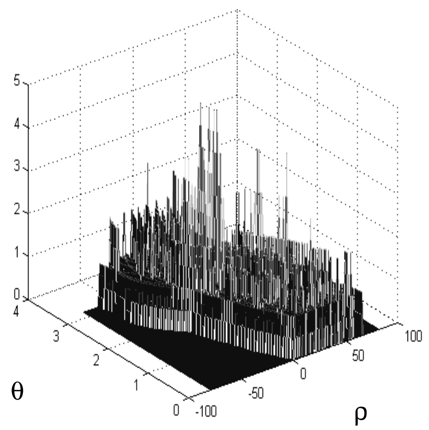

Figure 6. Parameter space histogram

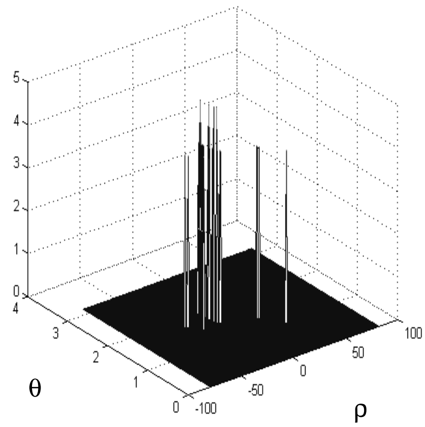

Figure 7. Parameter space after threshold detection

The efficiency of preliminary ghost elimination technique in the same case is demonstrated on figure 3, figure 8 and figure 9. As a result all ghosts were eliminated and the number of potential tracks was reduced.

The scenario of two tracks observed by three sensors was simulated. Every pair of sensors processes separately the scenario using the preliminary technique. The combined picture of FS is displayed on figure 4, where it can be seen that not all ghost intersection have been rejected. But, if the three PS, obtained from HT are combined (figure 5) the true target trajectories stand out definitely against the others.

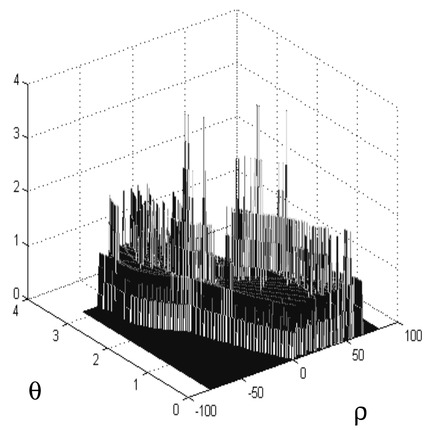

Figure 8. Parameter space histogram

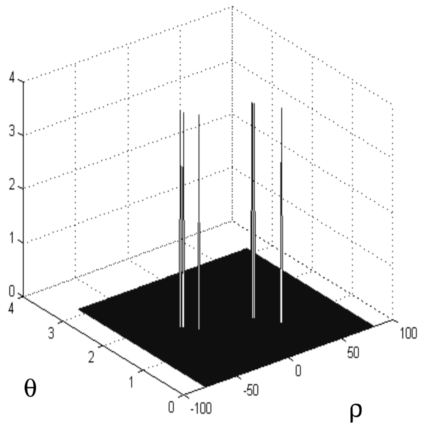

Figure 9. Parameter space after threshold detection

6 Conclusion

The paper concerns the dynamic data association problem for multiple passive sensors with known positions and in the presence of false alarms. The Hough transform is used to resolve the ambiguous measurement-target associations and to reject trajectories of ghosts in the case of two and more sensors. The main advantage of this approach is that in the case of three sensors data association problem solved by means of HT is not NP hard task. A heuristic ghost elimination technique is also proposed in the paper. This technique in addition increases the probability of recognizing false tracks, consisting of ghosts especially for two-sensor case. Numerical results are also presented.

7 References

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